

# Vector Theory of Gravity with Substratum

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A vector theory of gravity with substratum is proposed, which reproduces all the empirically verified linear and nonlinear predictions of Einstein's gravitational field equations, but unlike Einstein's theory, it does not lead to the unphysical singularities and causality-violating solutions permitting travel back in time. In the proposed alternative theory, a substratum or ether is the cause of all relativistic effects, and for this reason is assumed to obey a nonrelativistic equation of motion, an assumption which allows the substratum to consist of densely packed positive and negative Planck mass particles. A substratum of this kind can produce the vacuum fluctuations, needed to be in agreement with the empirical evidence, but avoids the paradox of infinite gravitational forces, predicted by a divergent zero point energy, the only one possible under the strict adherence to the kinematic interpretation of special relativity.

## 1. Introduction

There are two unresolved fundamental problems in gravitational physics: Certain unphysical solutions of Einstein's gravitational field equations, and the divergent zero point energy of relativistic quantum field theories. Einstein's gravitational field equations have two kinds of unphysical solutions: Solutions leading to unavoidable singularities in the course of gravitational collapse, and the causality-violating solutions permitting travel back in time, the latter produced by rotating masses. With regard to the zero point vacuum energy, there can be little doubt about its physical reality, manifesting itself in such a fundamental phenomenon as spontaneous emission, but an infinite zero point energy would result in infinite gravitational forces, in gross disagreement with the empirical evidence.

To overcome the problem of the infinite zero point energy, the suggestion is sometimes made that it has a natural cutoff at the Planck scale. As Hawking [1] has shown, the quantization of Einstein's field equations leads to a vacuum densely filled with Planck-mass black holes. Even though the unphysical infinity is thereby removed it still would lead to a mass density of the vacuum, equal to  $\sim 10^{94}$  g/cm<sup>3</sup>, large enough to put the mass of the entire known universe into a cube with a side length less than one fermi. It was remarked by Wheeler [2] that the negative energy of the gravita-

tional field set up in between the Planck black holes has the right magnitude to cancel the positive energy of the Planck masses, but this cannot resolve the problem because of the different form of the energy spectrum for the positive zero point energy, and the negative gravitational energy. Whereas, the zero point energy has a  $\omega^3$  frequency spectrum\*, the gravitational energy follows an  $\omega^5$  law. As a result, a compensation of both energies could only take place in the vicinity of the upper frequency cutoff at the Planck scale, leaving uncompensated the still huge zero point energy for all the lower frequencies.

We believe a resolution of these unresolved problems may come from questioning the ultimate correctness of special relativity. It is at the root of the infinite zero point vacuum energy, but also at other divergencies in relativistic quantum field theories. The unphysical divergences in relativistic quantum field theories and the unavoidable singularities in the solutions of Einstein's gravitational field equations may not be unrelated to each other and may rather have their common cause in a breakdown of special relativity at very high energies. Sure enough, special relativity even if ultimately false, would still be an extremely good approximation, like Newton's mechanics, which is an extremely good approximation of quantum mechanics for macroscopic bodies.

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\* This is the only relativistically invariant spectrum, with the Doppler- and aberration effect cancelling out against each other under a Lorentz transformation.

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The reason for questioning the ultimate truth of special relativity derives from a small sidereal tide measured with a superconducting gravimeter [3]. This otherwise unexplained tide can be interpreted as a nonadiabatic relativity-violation effect in the framework of the older Lorentz-Poincaré ether theory of relativity [4]. In this theory, all relativistic effects are explained as being caused by just one effect and which is a true Lorentz contraction suffered by all bodies in absolute motion through a substratum or ether. Because there the Lorentz contraction is real, it would need a finite time, and for times shorter, special relativity would be violated. For elementary particles this time should be extremely small, and which would explain why elementary particles satisfy the laws of relativity so well, but for macroscopic bodies the same would not be true. If interpreted as a non-adiabatic relativity-violating effect resulting from a finite contraction time, the observed sidereal tide would be consistent with an “ether wind” of about 300 km/sec, in remarkably good agreement with the velocity against the cosmic microwave background. However, because of its smallness and also because it has not been duplicated, different nonadiabatic experiments are needed. They may involve physical deformations through bending waves [5] ideally to be carried out in space.

In the dynamic version of the special theory of relativity by Lorentz and Poincaré, the Lorentz contraction is explained by the hypothesis that all physical bodies, including all elementary particles, are held together by electromagnetic forces or forces of the same kind, and that the equations governing these forces, which are Maxwell’s equations, are valid only in a reference system at rest with the substratum. The empirically established fact that Maxwell’s equations are also valid in any other inertial reference system is there explained as an illusion caused by a true Lorentz contraction of the observer and all his instruments. A violation of special relativity would be carried over into general relativity, which relates to transformations in between accelerated frames of reference, and from there on through the principle of equivalence to the theory of the gravitational field.

The ether abolished by Einstein as an unnecessary hypothesis reentered physics through quantum mechanics in form of the zero point energy of the vacuum. It is, therefore, not unreasonable to

ask if this zero point energy is the ether, or substratum, postulated in the dynamic interpretation of relativity by Lorentz and Poincaré. In the framework of quantum mechanics we can go one step further and ask about the true nature of this ether, that is of what it is composed and in which quantum mechanical state it is, and finally, what this ether if at all, might have to do with gravity.

## 2. Hypothesis About the Physical Nature of the Substratum

In an unsuccessful attempt to remove the well-known divergencies from relativistic quantum field theories, Dirac [6] tried to reintroduce the ether concept into physics. Several years later, Sakharov [7] made an attempt to introduce the zero point vacuum energy into Einstein’s gravitational field equations with a cutoff at the Planck length, but without reaching any definite conclusion. Common to both approaches was the assumption that the ether is subject to the laws of relativity. As we believe, this may have been the reason why these attempts failed. Because if the ether is the cause of all the relativistic effects, it should not be subject to the laws of relativity. The only reasonable alternative left open, is that it should instead exactly obey the nonrelativistic laws of Newtonian mechanics.

The conjecture that the ether obeys the laws of nonrelativistic mechanics admits the hypothesis that it consists of a densely packed assembly of fundamental particles of positive and negative mass. Their total numbers are conserved, with the number operator commuting with the Hamilton operator. The fundamental constants of physics, furthermore, suggest that the diameter of these particles is equal to the Planck length, and that the absolute value of their mass is equal to the Planck mass. For this reason we may simply call them Planckions. The zero point energy spectrum of each particle assembly would roughly resemble a Debye spectrum with a cutoff at the Planck scale. If there is an equal number of positive and negative mass Planckions, a vacuum filled with such a medium would still possess the quantum fluctuations needed to be in agreement with the empirical evidence, but it would not lead to infinite (or very large) mass densities and gravitational fields, and which are contradicted by the empirical evidence.

With the Planck mass as the upper absolute mass occupying a small volume having the diameter of the Planck length, Planckions of equal sign would be impenetrable. In contrast, two Planckions of opposite mass sign could completely penetrate and compensate each other without violating the rule for an upper mass within a smallest volume determined by the Planck length. The ether or substratum made up of the two species of Planckions therefore has the strange property that with regard to each separate species it would be incompressible, but combined completely compressible, and there would be no forces derived from a pressure gradient. The equation of state for the substratum, therefore, would be

$$\langle p \rangle = \langle \varrho \rangle = 0 \quad (2.1)$$

with the values of  $\langle p \rangle$  and  $\langle \varrho \rangle$  to be understood as time averages, and with the quantum mechanical fluctuations going all the way up to the Planck energy.

In a compressed substratum, Planckions of opposite mass would overlap and cancel each other, with the result that a more compressed substratum would be indistinguishable in its appearance from a less compressed one. Likewise, an expansion of the substratum would produce new pairs of Planckions with opposite mass, filling any void which is created by the expansion. The pairs can thereby be seen as having already existed before the expansion, hidden in an overlapping configuration. We therefore can say that in any reference system at rest with the substratum it always would appear the same.

If both positive and negative mass components of the substratum are in a quantum mechanical ground state, we may assume that both are superfluid. Combined with the smallness of the Planck length, the substratum should therefore be able to flow without resistance through all matter\*.

### 3. Gravitational Field Equations

The success of gauge theories suggests that all forces of nature, not just the electroweak inter-

actions, might be communicated by vector gauge bosons. Attempts made in the past to formulate vector theories of gravity in analogy to electrodynamics, had to fail because they would lead to a repulsion, not an attraction. A very different situation exists for vector theories with a substratum, in which negative values for the electric and magnetic permeabilities can reverse a repulsion between equal charges into an attraction. A simple procedure like that, however, is insufficient in formulating a vector theory of gravity, the reason being the nonlinearity of the gravitational field, but in a substratum theory nonlinearities can enter through the nonlinear equations governing the substratum fluid. This happens, for example, in magnetohydrodynamics or plasma physics, where the linear Maxwell equations are to be combined with the nonlinear Euler equations. A different problem for a vector theory with a substratum is the tensorlike behavior of the gravitational field, empirically confirmed through the energy loss of a double star pulsar, but in a vector substratum theory tensorlike waves can arise as in an elastic body, where the fundamental forces are still communicated by the vector gauge bosons of the electromagnetic field. It, therefore, appears that a vector theory with substratum has all the symmetries needed to describe gravity.

In Einstein's gravitational field theory the entire stress-energy-momentum tensor acts as a source of the field, whereas in a vector theory the source would be restricted to the energy momentum four vector. Because of the smallness of the predicted effects, there is no empirical evidence which proves that stresses or pressures are a source of the gravitational field, but there are sound physical reasons to doubt that this is the case. It is not denied that a body deformed under pressure or stress would have its rest mass changed due to the elastic potential energy, and therefore would also have its gravitational mass changed by the same amount, but that pressures or stresses are an unlikely source of a gravitational field can be illustrated by the simple example of a body which is subjected to a pressure  $p$ . As a result of the pressure applied its volume changes from  $V = V_0$  to  $V = V_1$ , and it gains an amount of energy and hence mass which is  $\Delta m =$

$$-(1/c^2) \int_{V_0}^{V_1} p \, dV. \text{ However, no such change would}$$

\* The conjecture that the ether has zero viscosity was first made by Helmholtz, prior to the discovery of quantum mechanics and superfluidity.

occur for an incompressible fluid where  $V_1 = V_0$ , but in which the pressure can assume any value. For an incompressible fluid the pressure gradient force is a constraint force and it is difficult to believe that a constraint could become the source of a gravitational field. No such logical contradiction can arise in a vector theory of gravity where the only source of the field is the energy-momentum four vector.

In keeping with the electrodynamic analogy, the vector theory of gravity should be described by a linear set of field equations. This means that the gravitational field itself shall have no gravitational mass, even though it would still possess inertial mass and energy. The principle of equivalence would, therefore, not be valid for the gravitational field. If it would be otherwise, the field equations would become nonlinear. The gravitational field is assumed to set into motion the substratum, and it is through the interaction of the flowing substratum with bodies placed in it that the gravitational forces actually observed, including their nonlinear properties, are generated. The principle of equivalence empirically established, is now explained by the almost trivial fact that the inertial forces acting on bodies placed in an accelerated frame of reference in relative motion against the substratum, must be like the forces if the bodies are unaccelerated but placed in a substratum which is set into motion by a gravitational field.

To formulate the gravitational field equations, we assume that the substratum acts like a medium for which the electric and magnetic permeability are  $\varepsilon = \mu = -1$ , and that the source of the gravitational field is the special-relativistic four-current vector of matter. The gravitational field equations for the scalar potential  $\Phi$  and vector potential  $A$  therefore are ( $\kappa = \text{const}$ )

$$\begin{aligned}\square \Phi &= \kappa \frac{\varrho_0}{1 - V^2/c^2}, \\ \square A &= \kappa \frac{\varrho_0 V/c}{1 - V^2/c^2}.\end{aligned}\quad (3.1)$$

In these equations  $\varrho_0$  is the rest mass density of the sources as they would appear in the absence of a gravitational field and  $V$  their velocity. The substratum, consisting of positive and negative mass Planckions, is not a source of the gravitational field, because the time average of its mass density

is zero. Equations (3.1) have to be supplemented by a Lorentz gauge condition

$$\frac{1}{c} \frac{\partial \Phi}{\partial t} + \text{div} A = 0. \quad (3.2)$$

The factor  $(1 - V^2/c^2)^{-1}$  is the product of two equal factors. One accounts for the relativistic mass increase with velocity, and the other one for the relativistic reduction of the volume element. Both combined increase the density by the factor  $(1 - V^2/c^2)^{-1}$ . From the demand that in the Newtonian limit ( $G$  Newtonian gravitational constant)

$$\nabla^2 \Phi = 4\pi G \varrho, \quad (3.3)$$

we find  $\kappa = 4\pi G$ . The adjustment of the vector potential will be made below.

Inserting (3.1) into the gauge condition we obtain the relativistic continuity equation

$$\frac{\partial}{\partial t} \left( \frac{\varrho_0}{1 - V^2/c^2} \right) + \text{div} \left( \frac{\varrho_0 V}{1 - V^2/c^2} \right) = 0. \quad (3.4)$$

In Lagrangian form the Newtonian equation of the substratum is

$$\begin{aligned}\frac{dv}{dt} &= -\frac{\alpha}{c} \frac{\partial A}{\partial t} - \nabla \Phi \\ &\quad + \frac{\alpha}{c} v \times \text{curl} A,\end{aligned}\quad (3.5)$$

which in Eulerian form is

$$\begin{aligned}\frac{\partial v}{\partial t} + (v \cdot \nabla) v \\ = -\frac{\alpha}{c} \frac{\partial A}{\partial t} - \nabla \Phi + \frac{\alpha}{c} v \times \text{curl} A.\end{aligned}\quad (3.6)$$

The still to be determined factor  $\alpha$  measures the coupling strength of the vector potential to the substratum. A more convenient form of (3.6) is

$$\begin{aligned}\frac{\partial v}{\partial t} + \frac{1}{2} \nabla v^2 - v \times \text{curl} v \\ = -\frac{\alpha}{c} \frac{\partial A}{\partial t} - \nabla \Phi + \frac{\alpha}{c} v \times \text{curl} A.\end{aligned}\quad (3.7)$$

We split the substratum velocity into two parts:

$$v = v_{(1)} + v_{(2)}, \quad (3.8)$$



with the two parts having the property that

$$\left. \begin{aligned} \text{curl } \mathbf{v}_{(1)} &= 0, \\ \text{div } \mathbf{v}_{(2)} &= 0. \end{aligned} \right\} \quad (3.9)$$

At this point, we take notice that a gravitational field described by these two three-dimensional velocity fields depends on as many independent space-time functions as Einstein's theory. The two velocity fields are described by six functions, the same number as in Einstein's theory, where the ten components of the metric tensor are restricted by four gauge conditions.

To determine  $\alpha$ , and hence the coupling strength of the gravitational vector potential  $\mathcal{A}$  with the substratum, we consider the equation of motion for a test particle placed in the gravitational field. If this field is described by a line element  $ds^2 = g_{ik} dx^i dx^k$ , the equation of motion can be obtained from the variational principle

$$\delta \int ds = 0. \quad (3.10)$$

The equation of motion for the substratum, and which shall obey the nonrelativistic Newtonian law, can be obtained from the variational principle (3.10) as well, putting everywhere  $v^2/c^2 = 0$ , but keeping terms of the order  $v/c$ . This amounts in taking the nonrelativistic limit. In this limit the gravitational field interacting with the substratum can always be considered weak.

We can write the line element valid for the motion of the substratum as

$$\begin{aligned} ds^2 &= (\delta_{ik} + \gamma_{ik}) dx^i dx^k \\ &= \left( v^2 - c^2 + \gamma_{ik} \frac{dx^i}{dt} \frac{dx^k}{dt} \right) dt^2, \end{aligned} \quad (3.11)$$

where  $x^4 = ict$ . Because  $ic ds = L dt$ , where  $L$  is the Lagrange function per unit mass, we have

$$L = -c^2 \sqrt{1 - \frac{v^2}{c^2} - \frac{\gamma_{ik}}{c^2} \frac{dx^i}{dt} \frac{dx^k}{dt}}. \quad (3.12)$$

To obtain the nonrelativistic limit, we first expand (3.12) (omitting the additive constant  $-c^2$ ):

$$L = \frac{v^2}{2} + \frac{\gamma_{ik}}{2} \frac{dx^i}{dt} \frac{dx^k}{dt}. \quad (3.13)$$

The terms  $\gamma_{ik}$ ,  $i, k = 1, 2, 3$  are of the order  $v^2/c^2$ . Therefore, taking in (3.13) only terms up to the

first order in  $v/c$ , we finally obtain the Newtonian limit

$$L = \frac{v^2}{2} + ic \gamma_{i4} v^i - \frac{c^2}{2} \gamma_{44}. \quad (3.14)$$

The curvilinear coordinate system in which the line element (3.11) is expressed, is not unique. It is subject to four arbitrary functions which allow transformations to other curvilinear coordinate systems, including those involving accelerated frames of reference. Since in curvilinear coordinate systems inertial forces do occur and which mimic gravitational fields, a procedure is therefore needed to find a coordinate system from which these inertial forces are eliminated. In a Riemannian space-time such a coordinate system does not exist. However, even in a Riemannian space a curvilinear coordinate system exists for which the inertial forces are minimized. It is a coordinate system which comes as close as possible to a four-dimensional Cartesian coordinate system and is determined by the DeDonder-condition [8]

$$g^{lm} \Gamma_{lm}^i = 0. \quad (3.15)$$

Whereas in a Riemannian manifold it is in general not possible to choose a coordinate system from which the inertial forces are completely eliminated, this is possible in case the departure from a Euclidean metric is small. There the space-time can either be described as slightly non-Euclidean, with a line element given by (3.11), or by a perfectly Euclidean manifold with a linear tensor field  $\psi_{ik}$ . In case of a space-time described by a linear tensor-field, (3.15) becomes

$$\partial \psi_{ik} / \partial x^k = 0 \quad (3.16)$$

$$\begin{aligned} \text{with } \psi_{ik} &\equiv \gamma_{ik} - \frac{1}{2} \delta_{ik} \gamma, \\ \gamma &\equiv \gamma_i^i. \end{aligned} \quad (3.17)$$

From (3.17) with  $\psi_i^i = \psi = -\gamma_i^i = -\gamma$  we have

$$\gamma_{ik} = \psi_{ik} - \frac{1}{2} \delta_{ik} \psi; \quad (3.18)$$

$$\text{hence } \gamma_{i4} = \psi_{i4}, \quad \gamma_{44} = \frac{1}{2} \psi_{44}. \quad (3.19)$$

The Lagrange function (3.14) therefore becomes

$$L = \frac{v^2}{2} + ic \psi_{i4} v^i - \frac{c^2}{4} \psi_{44}. \quad (3.20)$$

If we put

$$ic^2\psi_{i4}/4 = A_i, \quad c^2\psi_{44}/4 = \Phi, \quad (3.21)$$

(3.16) for  $i = 4$  becomes the gauge condition

$$\operatorname{div} \mathbf{A} + \frac{1}{c} \frac{\partial \Phi}{\partial t} = 0. \quad (3.22)$$

In terms of  $\mathbf{A}$  and  $\Phi$  the Lagrange function (3.20) finally reads

$$L = v^2/2 + 4A_i v^i/c - \Phi. \quad (3.23)$$

With the Lagrange function (3.23), one obtains the equation of motion for the substratum in Lagrangian form if  $\alpha = 4$ .

Even though the fourfold larger force of the vector potential  $\mathbf{A}$ , expressed by setting  $\alpha = 4$  in (3.5), resp. (3.7), destroys the general gauge invariance of the theory, we still can make the gauge transformation

$$\left. \begin{aligned} \mathbf{A}' &= \mathbf{A} + \frac{1}{4} \nabla f, \\ \Phi' &= \Phi - \frac{1}{c} \frac{\partial f}{\partial t}, \end{aligned} \right\} \quad (3.24)$$

which keeps (3.5), resp. (3.7) invariant. However, the Lorentz gauge condition (3.2) can then in general not be satisfied under this change, because  $f$  must satisfy the wave equation  $-(1/c^2) \partial^2 f / \partial t^2 + \nabla^2 f = 0$ . This result is not surprising if we realize that in a gravitational field theory the potentials must have an absolute meaning. For stationary gravitational fields their value is determined from the boundary condition that at  $r = \infty$ ,  $\Phi(\infty) = \mathbf{A}(\infty) = 0$ . For stationary fields, (3.7) has under these boundary conditions the solution

$$\left. \begin{aligned} v^2 &= (v_{(1)} + v_{(2)})^2 = -2\Phi, \\ v_{(2)} &= -(4/c)\mathbf{A}. \end{aligned} \right\} \quad (3.25)$$

The solution of the gravitational field equations, however, must still be invariant under a Lorentz transformation. This means it has to be invariant under the addition of a constant velocity  $v_0$ , by which

$$f = -c v_0 \cdot \mathbf{r} + \frac{1}{2} c v_0^2 t \quad (3.26)$$

and for which  $\nabla^2 f = \partial^2 f / \partial t^2 = 0$ , identically satisfying the wave equation for  $f$ .

Under the transformation (3.24) we thus have

$$\left. \begin{aligned} \mathbf{A}' &= \mathbf{A} - \frac{1}{4} c v_0 \\ \Phi' &= \Phi - \frac{1}{2} v_0^2. \end{aligned} \right\} \quad (3.27)$$

A Lorentz transformation has the effect that all measuring devices would be altered in such a way as to make the addition of a constant velocity undetectable, and which permits to put  $v_0 = 0$ .

For the general time dependent case, the potentials can be uniquely determined by the retarded potential\* solution of (3.1) together with the gauge condition (3.2), provided the sources of all fields are known. It is, however, conceivable that there are gravitational waves traversing the universe which cannot be traced to a source. If such "wandering" gravitational waves should exist, something which can be only established by observation, the potentials could still be uniquely determined by a linear combination of the retarded potential solution of (3.1) for the known sources, and a solution of the corresponding homogeneous equations. The observational data deduced from the wandering waves would thereby determine the initial conditions to arrive at a unique solution for the potentials. Since it appears doubtful that such sourceless wandering waves exist, the potentials might always be uniquely determined by the retarded potential solution alone.

We note that the scalar potential always leads to two signs in the velocity  $v$ , by the taking of the square root of  $-2\Phi$  in (3.25). These two signs corresponds to the inflowing and outflowing substratum attracted towards and passing through a field source. The vector potential leads to only one direction of the substratum velocity vector  $v_{(2)}$ .

#### 4. The Principle of Equivalence

From our version of the principle of equivalence we can determine the forces acting on a body if placed in the flowing substratum set into motion through a gravitational field, by comparing them with the inertial forces which result if the body is placed in an accelerated frame of reference, relative to which the substratum is in motion.

\* We take the position that the advanced potential solutions have no physical meaning and are, therefore, to be rejected.

From the vector form of the Lorentz transformations,

$$\mathbf{r}' = \mathbf{r} + \mathbf{v} \left[ \frac{\mathbf{r} \cdot \mathbf{v}}{v^2} \left( \frac{1}{\sqrt{1-v^2/c^2}} - 1 \right) - \frac{t}{\sqrt{1-v^2/c^2}} \right], \quad (4.1)$$

$$t = \frac{t' + (\mathbf{r}' \cdot \mathbf{v})/c^2}{\sqrt{1-v^2/c^2}}, \quad (4.2)$$

where we assumed that the unprimed reference system is at rest with the substratum, we find that for a fixed position in time  $t = \text{const}$ , and for a fixed position in space,  $\mathbf{r}' = \text{const}$ ,

$$d\mathbf{r}'^2 = d\mathbf{r}^2 + \frac{(\mathbf{v} \cdot d\mathbf{r})^2}{c^2 - v^2}, \quad (4.3)$$

$$dt' = \sqrt{1-v^2/c^2} dt. \quad (4.4)$$

For the special case where  $\mathbf{v}$  is aligned with  $d\mathbf{r}$ , we obtain from (4.3) the one-dimensional Fitzgerald-Lorentz contraction formula  $d\mathbf{r} = d\mathbf{r}' \sqrt{1-v^2/c^2}$ . Likewise from (4.4) follows the usual time dilation formula  $dt = dt'/\sqrt{1-v^2/c^2}$ . Equations (4.3) and (4.4) are an expression of how the motion of the substratum in the primed system affects rods and clocks.

The transition from an unaccelerated to an accelerated frame of reference is straightforward. First, we may assume that at some specified time the accelerated frame is at rest with regard to the unaccelerated frame of reference. We can then use this moment to synchronize clocks positioned in both systems. To keep all clocks thereafter synchronized, we must only let all the clocks in the accelerated frame run in each moment faster by the factor  $1/\sqrt{1-v^2/c^2}$ , where  $v$  is the velocity attained due to the acceleration, and which can be determined by integrating the reading of an accelerometer.

The transition from the unaccelerated frame  $\mathbf{r}_0, t_0$ , and which we may assume is at rest with the substratum, to the accelerated frame  $\mathbf{r}, t$ , is then given by the equations

$$\left. \begin{aligned} d\mathbf{r}_0 &= d\mathbf{r} + (\mathbf{v} + \boldsymbol{\omega} \times \mathbf{r}) dt, \\ dt_0 &= dt. \end{aligned} \right\} \quad (4.5)$$

In (4.5)  $\mathbf{v}$  is the translational and  $\boldsymbol{\omega}$  the angular velocity of the  $\mathbf{r}, t$ , frame of reference. Clearly,

$$\mathbf{v} = -(\mathbf{v} + \boldsymbol{\omega} \times \mathbf{r}) \quad (4.6)$$

is the velocity of the substratum at some point in the accelerated frame.

The transformations (4.5) change the line element in the substratum rest frame

$$ds_0^2 = d\mathbf{r}_0^2 - c^2 dt^2 \quad (4.7)$$

into the line element in the accelerated frame:

$$ds^2 = ds_0^2 - 2\mathbf{v} \cdot d\mathbf{r} dt + v^2 dt^2. \quad (4.8)$$

In the following step we split the line element in its space and time parts [9]. If the line element has the general form  $ds^2 = g_{ik} dx^i dx^k$ ,  $i, k = 1, 2, 3, 4$ , its space part is (with  $\mu, \nu = 1, 2, 3$ )

$$d\sigma^2 = \left( g_{\mu\nu} + \frac{g_{\mu 4} g_{\nu 4}}{-g_{44}} \right) dx^\mu dx^\nu, \quad (4.9)$$

and the time part

$$d\tau^2 = -g_{44} dt^2. \quad (4.10)$$

It is convenient to introduce the vector  $\mathbf{g}$  with the components  $g_{\mu 4}$ . The components of the metric tensor for the line element (4.8) are then

$$\left. \begin{aligned} g_{\mu\nu} &= \delta_{\mu\nu}, \\ \mathbf{g} &= -\mathbf{v}, \\ -g_{44} &= 1 - v^2/c^2 \end{aligned} \right\} \quad (4.11)$$

hence

$$ds^2 = d\mathbf{r}^2 + 2\mathbf{g} \cdot d\mathbf{r} dt - (c^2 - v^2) dt^2 \quad (4.12)$$

and therefore

$$\left. \begin{aligned} d\sigma^2 &= d\mathbf{r}^2 + \frac{(\mathbf{v} \cdot d\mathbf{r})^2}{c^2 - v^2}, \\ d\tau^2 &= (1 - v^2/c^2) dt^2, \end{aligned} \right\} \quad (4.13)$$

in full agreement with (4.3) and (4.4). Therefore, the effect of the substratum motion on rods and clocks in an accelerated frame of reference, as in the ether interpretation of special relativity, is uniquely determined by the motion of the substratum. The substratum velocity in the accelerated reference system here too causes the rod contraction and time dilation effects. In addition though, due

to the non-Galilean form of the metric expressed by the line element (4.8), inertial forces occur and which are proportional to the Christoffel-symbols computed from the metric tensor (4.11) for this line element. Expressed in vector form, these inertial forces are

$$\mathbf{f} = -\varrho[\dot{\mathbf{v}} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \dot{\mathbf{r}}]. \quad (4.14)$$

In this equation  $\varrho$  is the density of a test body and  $\mathbf{r}$  its position. Through (4.6) the inertial forces can be expressed in terms of the substratum velocity  $\mathbf{v}$ :

$$\mathbf{f}_\mu = \varrho \left[ \frac{\partial v_\mu}{\partial t} + \frac{\partial}{\partial x^\nu} (v_\mu v_\nu) - (\mathbf{V} \times \text{curl } \mathbf{v})_\mu \right] \quad (4.15)$$

with  $\dot{\mathbf{r}} = \mathbf{V}$  the velocity of the test body. In deriving (4.15) the property  $\text{div } \mathbf{v} = 0$  was used by which

$$\partial/\partial x^\nu (v_\mu v_\nu) = v_\nu \partial v_\mu / \partial x^\nu.$$

In the Lorentz-Poincaré view the inertial forces must be seen as being caused by a stress resulting from a strain of bodies placed in the flowing ether of the accelerated reference system. The spatial line element (4.9), explicitly given by

$$d\sigma^2 = \left( \delta_{\mu\nu} + \frac{v_\mu v_\nu / c^2}{1 - v^2/c^2} \right) dx^\mu dx^\nu, \quad (4.16)$$

can be interpreted as an expression for a strain tensor [10], which is

$$\varepsilon_{\mu\nu} = \frac{1}{2} \frac{v_\mu v_\nu / c^2}{1 - v^2/c^2}. \quad (4.17)$$

Assuming that the stress-strain relation is

$$\sigma_{\mu\nu} = \varrho c^2 \varepsilon_{\mu\nu}, \quad (4.18)$$

the second term in the bracket of the r.h.s. of (4.15), and which is the centrifugal force, is in the nonrelativistic limit recovered in the form

$$\frac{\partial \sigma_{\mu\nu}}{\partial x^\nu} = \varrho \frac{\partial}{\partial x^\nu} (v_\mu v_\nu). \quad (4.19)$$

The third term in the bracket on the r.h.s. of (4.15), the Coriolis force, can be understood as a Magnus-type force\*

$$\mathbf{f}_M = -\varrho \mathbf{V} \times \text{curl } \mathbf{v}. \quad (4.20)$$

\* In fluid dynamics known as the Kutta-Joukowski theorem.

Finally, the first term

$$\mathbf{f}_D = \varrho \frac{\partial \mathbf{v}}{\partial t}, \quad (4.21)$$

which is the “ordinary”\*\* inertial force, can be interpreted as the inertial drag by the flowing substratum, in complete analogy to the inertial drag in fluid dynamics.

An important remark is here in order. In Einstein’s view the metric expressed by the components of the metric tensor (4.11) is flat, because the Riemann tensor computed from this metric tensor vanishes. In the Lorentz-Poincaré view this has a somewhat different reason. It there follows from the particular form of the ether velocity (4.6), for which  $\text{div } \mathbf{v} = 0$ , and  $\text{curl } \mathbf{v} = -2\boldsymbol{\omega}$ , where  $\boldsymbol{\omega}$  is a (in general time dependent) vector constant in space. In contrast to gravitational fields mimicked by inertial forces in accelerated frames of references, this property is not carried over in the presence of true gravitational fields. However, regardless whether  $\text{div } \mathbf{v} = 0$  and  $\text{curl } \mathbf{v} = \text{constant}$  in space or not, in the Lorentz-Poincaré view the more fundamental physical effect is always a true deformation of physical bodies in their absolute motion against the ether. To express this in terms of a (in general non-Euclidean) metric is in the context of the Lorentz-Poincaré ether theory to be seen only as a convenient way to describe this complex physical situation.

To see how the two velocity fields  $\mathbf{v}_{(1)}$ , and  $\mathbf{v}_{(2)}$ , for which  $\text{curl } \mathbf{v}_{(1)} = 0$  and  $\text{div } \mathbf{v}_{(2)} = 0$ , determine the metric tensor, we rewrite the line element (4.12) in a form which separates its space and time part from the rest. Using (4.13) we have

$$ds^2 = d\sigma^2 - \frac{(\mathbf{g} \cdot d\mathbf{r})^2}{c^2 - v^2} + 2\mathbf{g} \cdot d\mathbf{r} - c^2 d\tau^2. \quad (4.22)$$

To make the transition from inertial force fields (for which always  $\mathbf{v}_{(1)} = 0$  and  $\mathbf{v} = \mathbf{v}_{(2)}$ ) to true gravitational fields, we substitute wherever  $v^2$  occurs its total value (3.8). Therefore, in making this substitution, the space and time parts of  $ds^2$ , as

\*\* The terminology “ordinary” inertial force is here used, because (4.14), can be seen as the covariant derivative of the velocity with regard to the time, possessing an ordinary and covariant part, with the covariant part containing the contributions coming from the Christoffel-symbols.



they are given by (4.13), are retained. And the same substitution must be made in the denominator of the term  $(\mathbf{g} \cdot d\mathbf{r})/(c^2 - v^2)$ . No such simple assignment exists for the vector  $\mathbf{g}$ . As we would expect, it is related to the vorticity of the substratum flow. In general relativity (by which we mean general curvilinear coordinate transformations in between space and time coordinates), this substratum vorticity is there rather seen as an intrinsic rotation against an inertial frame of reference, with the rotational velocity equal and opposite to the substratum velocity. An expression for this intrinsic rotation in terms of a local angular velocity vector  $\boldsymbol{\omega}$  has been given by von Weyssenhoff [11]. Applied to our line element (4.22), (in conjunction with the chosen clock synchronization for arbitrarily accelerated frames of reference, explained above in connection with the transformation (4.5)), this expression is

$$\boldsymbol{\omega} = \frac{1}{2} \left[ \text{curl} \mathbf{g} - \frac{\mathbf{g}}{c^2} \times \frac{\partial \mathbf{g}}{\partial t} \right]. \quad (4.23)$$

Therefore, setting  $\text{curl} \mathbf{v}_{(2)} = -2\boldsymbol{\omega}$ , a relation in between  $\mathbf{g}$  and  $\mathbf{v}_{(2)}$  is obtained:

$$\left[ \text{curl} \mathbf{g} - \frac{\mathbf{g}}{c^2} \times \frac{\partial \mathbf{g}}{\partial t} \right] = -\text{curl} \mathbf{v}_{(2)}. \quad (4.24)$$

To test the correctness of this expression we apply it to a co-rotating frame of reference. According to (4.6) one there has  $\mathbf{v} = \mathbf{v}_{(2)} = -\boldsymbol{\omega} \times \mathbf{r} = -\mathbf{g}$ , and for which (4.24) is, as expected, identically satisfied.

In general (4.24) represents three coupled partial differential equations, the solutions of which determine  $\mathbf{g}$  in terms of  $\mathbf{v}_{(2)}$ . It appears that only in the case where  $\partial/\partial t = 0$ , and for which the gravitational field is stationary, a simple solution exists. It is given by

$$\mathbf{g} = -\mathbf{v}_{(2)}. \quad (4.25)$$

This solution would normally include a gradient of some function to be added to it. However, since the curl of such a term always vanishes, it must be part of  $\mathbf{v}_{(1)}$ .

In case of stationary gravitational fields, we can write the line element as

$$ds^2 = d\sigma^2 - \frac{(\mathbf{v}_{(2)} \cdot d\mathbf{r})^2}{c^2 - v^2} - 2\mathbf{v}_{(2)} \cdot d\mathbf{r} dt - c^2 d\tau^2, \quad (4.26)$$

or more explicitly as

$$ds^2 = d\mathbf{r}^2 + \frac{(\mathbf{v} \cdot d\mathbf{r})^2 - (\mathbf{v}_{(2)} \cdot d\mathbf{r})^2}{c^2 - v^2} - 2\mathbf{v}_{(2)} \cdot d\mathbf{r} dt - (c^2 - v^2) dt^2. \quad (4.27)$$

## 5. The Equations of Motion

The equation of motion for a test particle in a gravitational field can be obtained from (3.10), using the expression of the metric tensor given by (4.27), in exactly the same way as it is done in Einstein's theory. However, if one wants to formulate the law in accordance with what we had found out about how the nonuniform substratum flow acts upon a body if placed in an accelerated frame of reference, we have to put

$$f_\mu = \frac{\partial \sigma_{\mu\nu}}{\partial x^\nu} + \left[ \left( \frac{V}{c} \right) \times \text{curl} \boldsymbol{\sigma} + \frac{1}{c} \frac{\partial \sigma_4}{\partial t} \right]_\mu. \quad (5.1)$$

There the stresses caused by the substratum flow are

$$\left. \begin{aligned} \sigma_{\mu\nu} &= \frac{\varrho}{2} \frac{v_\mu v_\nu}{1 - v^2/c^2}, \\ \boldsymbol{\sigma} &= -\varrho c \mathbf{v}_{(2)}, \\ \sigma_4 &= \varrho c v. \end{aligned} \right\} \quad (5.2)$$

In (5.2)  $\sigma_{\mu\nu}$  is a symmetric stress tensor,  $\boldsymbol{\sigma}$  a Coriolis stress vector and  $\sigma_4$  a drag stress vector.

For an irrotational substratum flow, describing a static gravitational field with  $\text{curl} \mathbf{v} = 0$  and  $\partial/\partial t = 0$ , one has from (3.25)

$$v^2/2 = -\Phi. \quad (5.3)$$

In the limit  $v \ll c$ , the component of the stress tensor  $\sigma_{\mu\nu}$  in the direction of  $\mathbf{v}$  is

$$\sigma = (1/2) \varrho v^2 = -\varrho \Phi. \quad (5.4)$$

The force in the direction of  $\mathbf{v}$  is then just Newton's law

$$\mathbf{f} = \nabla \sigma = -\varrho \nabla \Phi. \quad (5.5)$$

For a stationary (non-static) substratum flow, where only  $\partial/\partial t = 0$ , one has from (3.25)

$$\mathbf{v} = \mathbf{v}_{(2)} = -\frac{4}{c} \mathbf{A}. \quad (5.6)$$

This divergence-free substratum velocity field leads to a Magnus-force which is given by the second term in the expression for  $f_\mu$ :

$$\mathbf{f}_M = \frac{4\rho}{c} \mathbf{V} \times \text{curl} \mathbf{A}. \quad (5.7)$$

It is the gravitational analog to the Lorentz-force, but four times larger. It also describes the gravitational radiation pressure, because a gravitational wave can give a body a velocity  $\mathbf{V}$ , which thereafter interacts with the curl  $\mathbf{v}$ -term of the wave.

With the strain-stress relations given by

$$\left. \begin{aligned} \sigma_{\mu\nu} &= \rho c^2 \varepsilon_{\mu\nu}, \\ \sigma &= \rho c^2 \varepsilon, \\ \sigma_4 &= \rho c^2 \varepsilon_4 \end{aligned} \right\} \quad (5.8)$$

one has for the Lorentz-type deformations causing a strain:

$$\left. \begin{aligned} \varepsilon_{\mu\nu} &= \frac{1}{2} \frac{v_\mu v_\nu / c^2}{1 - v^2/c^2} \\ \varepsilon &= -v_{(2)}/c, \\ \varepsilon_4 &= v/c. \end{aligned} \right\} \quad (5.9)$$

Finally, we may introduce a stress in time defined as

$$\varepsilon_0 = (d\tau - dt)/dt = (\sqrt{1 - v^2/c^2} - 1) \quad (5.10)$$

with the corresponding stress-strain relation

$$\sigma_0 = \rho c^2 \varepsilon_0 = \rho c^2 (\sqrt{1 - v^2/c^2} - 1). \quad (5.11)$$

In a static gravitational field with  $(1/2)v^2 = -\Phi$ , one has up to order  $v^2/c^2$ :

$$\sigma_0 = \rho \Phi. \quad (5.12)$$

It there represents the change in energy of the body placed inside a gravitational field, due to its change in the gravitational potential.

Equation (5.11) explains why the mass density  $\rho_0$  in (3.1) and (3.4) must be the same as in the absence of gravitational fields, for the following reason: According to (5.11), the mass of a body placed in a gravitational field is reduced by the

factor  $\sqrt{1 - v^2/c^2}$ , but the volume changes by the same factor, thereby keeping the density unchanged. This explains why our assumption that the gravitational field is not a source to itself worked, because the gravitational mass of the field is accounted for in (3.1) through the deformation and change in size of the four-current sources by the flowing substratum. However, it must be kept in mind that this conclusion depends on the assumption of a linear strain-stress relationship, which is strictly valid only for infinitesimal deformations. Further studies must explore if under this assumption the energy-momentum conservation law can always be satisfied even for large deformations. Our approach is at least a very good approximation for the proposed physical model, because it reproduces the exact nonlinear Schwarzschild solution of Einstein's field equations.

## 6. Static Centrally Symmetric Gravitational Field

One principle question, of course, is if our alternative theory can explain, as well as Einstein's theory, the three empirically verified effects in a centrally symmetric gravitational field: The red shift, the light deflection and the perihelion motion. The easiest way to see that this is indeed the case, can be demonstrated by showing that the Schwarzschild line element can also be derived from our theory. It, however, must again be emphasized that the introduction of a metric is only a substitute for actual Lorentz-type deformations of bodies, which mimic a non-Euclidean Riemannian space-time for observers using such bodies to make measurements.

From (5.3) with  $v(\infty) = 0$ , one obtains for the radial substratum flow in the field of a mass  $M$

$$v_r^2 = \frac{2GM}{r}, \quad (6.1)$$

and hence from (4.27) in spherical polar coordinates the well-known Schwarzschild line element

$$\begin{aligned} ds^2 &= \frac{dr^2}{1 - \frac{2GM}{c^2 r}} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \\ &\quad - c^2 \left( 1 - \frac{2GM}{c^2 r} \right) dt^2. \end{aligned} \quad (6.2)$$

We will now show that the predicted three effects can also be derived from the stress forces by the substratum flow.

We first compute the perihelion motion. For a nearly circular orbit, Einstein's theory leads in the so-called post-Newtonian approximation to a perihelion shift which can be derived from the following equation of motion\*:

$$\ddot{r} = \left(1 + \frac{GM}{c^2 r}\right) GM \nabla \left(\frac{1}{r}\right). \quad (6.3)$$

Our theory leads to the same equation. According to (5.2), one obtains for the radial stress:

$$\sigma_{rr} = \frac{\varrho}{2} \left[ \frac{v_r^2}{1 - v_r^2/c^2} \right]. \quad (6.4)$$

The equation of motion of the body is then given by

$$\frac{d^2}{dt^2} \left[ \frac{r}{(1 - V^2/c^2)(1 - v_r^2/c^2)} \right] = \frac{\partial \sigma_{rr}}{\partial r}, \quad (6.5)$$

where  $V^2 = GM/r$ , with  $V$  the circular velocity of the planet in the centrally symmetric field. Inserting (6.4) into (6.5), we find after expanding up to terms of the order  $GM/c^2 r$  the post-Newtonian equation (6.3).

The deflection of light must be computed as a wave propagation phenomenon in the limit of ray optics. The flowing substratum produces a refractive index tensor  $n_{\mu\nu}$  and which is given by the ratio

$$n_{\mu\nu} = \frac{1 + \varepsilon_{\mu\nu}}{1 + \varepsilon_0}. \quad (6.6)$$

For a centrally symmetric field one has

$$\begin{aligned} n_{rr} &= \frac{1}{1 - 2GM/c^2 r} \simeq 1 + 2GM/c^2 r, \\ n_{\theta\theta} = n_{\varphi\varphi} &= \frac{1}{1 - GM/c^2 r} \simeq 1 + GM/c^2 r. \end{aligned} \quad (6.7)$$

This is the same as it would also follow from the Schwarzschild line element, and therefore leads to

the same value for the deflection of light as in Einstein's theory.

Finally, the red shift follows from the strain in time and is given by

$$\Delta\nu/\nu = \varepsilon_0 = \Phi/c^2 = -GM/c^2 r. \quad (6.8)$$

## 7. Other Solutions

Another problem of astrophysical importance is the gravitational field produced by a spinning sphere. It was first studied by Thirring and Lense [13] who obtained solutions from the linearized Einstein equations. An exact solution of Einstein's equations found by Kerr [14], goes for large distances over into the solution of Thirring and Lense. However, the Kerr solution can hardly be an exact solution for a spinning sphere, which by centrifugal forces would already in Newtonian mechanics assume a nonspherical shape.

In deriving an approximate, albeit nonlinear solution for a spinning sphere, we assume that its rotational speed is small and that it thereby retains its spherical shape. In addition to its Newtonian scalar potential it has a gravitational vector potential which at large distances is (computed from (3.1))

$$A_\varphi = -\frac{G}{2c} \frac{J \sin \theta}{r^2}, \quad (7.1)$$

where  $J$  is the angular momentum of the spinning sphere. The substratum velocity due to this vector potential therefore is

$$v_{(2)} = v_\varphi = -\frac{4}{c} A_\varphi = \frac{2G}{c^2} \frac{J \sin \theta}{r^2}. \quad (7.2)$$

If the gravitational vector potential is small compared to the Newtonian potential, we can approximately use for  $v_{(1)}$  the radial velocity  $v_r$  given by (6.1). Inserting  $v_{(1)}$  and  $v_{(2)}$  into the line element (4.27), we obtain the linearized field solution found by Thirring and Lense:

$$\begin{aligned} ds^2 &= \frac{dr^2}{1 - 2GM/c^2 r} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \\ &\quad - \frac{4GJ}{c^2 r} \sin^2 \theta d\varphi dt \end{aligned}$$

\* If the acceleration is measured with rods and clocks of the local reference system [12].

$$-c^2 \left(1 - \frac{2GM}{c^2 r}\right) dt^2. \quad (7.3)$$

However, we can do much better and obtain a nonlinear solution by solving (3.25) for  $v_{(1)}$  and  $v_{(2)}$ . In particular for  $\theta = 90^\circ$ , a solution in the equatorial plane can be obtained in closed form. There the angle in between  $v_{(1)}$  and  $v_{(2)}$  is  $90^\circ$ , and furthermore  $v_{(1)}$  has only a radial component. We there find

$$\begin{aligned} v_r^2 &= -2\Phi - \frac{16A_\varphi^2}{c^2} \\ &= \frac{2GM}{r} - \frac{4G^2J^2}{c^4 r^4} \end{aligned} \quad (7.4)$$

with  $v_\varphi$  unchanged, and therefore

$$\begin{aligned} ds^2 &= ds_0^2 + \frac{(v_r dr + v_\varphi r d\varphi)^2 - v_\varphi^2 r^2 d\varphi^2}{c^2 - v^2} \\ &\quad - 2v_\varphi r d\varphi dt + v^2 dt^2, \end{aligned} \quad (7.5)$$

where  $ds_0^2 = dr^2 + r^2 d\varphi^2 - c^2 dt^2$ .

The second term in (7.4) results in an induced centrifugal type force. In particular if  $v_r = 0$ , one obtains the radius at which the induced centrifugal-type field balances the Newtonian force. A compensation of this kind is believed to take place near a rotating black hole.

Of considerable interest is also gravitational radiation pressure, because the work done by this pressure is closely related to the damping by the emission of gravitational radiation. We assume a situation where in the absence of gravitational radiation  $v = 0$ , that is a situation where no static or stationary gravitational fields are present. The radiation pressure is then given by (5.7):

$$f_R = (4\varrho V/c) \times \text{curl} \mathbf{A}. \quad (7.6)$$

Assuming that  $|V| \ll c$ , we can express  $V$  through the wave equation (3.1) for the vector potential:

$$f_R = \frac{1}{\pi G} \left[ \nabla^2 \mathbf{A} - \frac{1}{c^2} \ddot{\mathbf{A}} \right] \times \text{curl} \mathbf{A}. \quad (7.7)$$

In analogy to electrodynamics we then put

$$\left. \begin{aligned} \mathbf{F} &= -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - \nabla \Phi, \\ \mathbf{B} &= \text{curl} \mathbf{A} \end{aligned} \right\} \quad (7.8)$$

and (using the gauge condition (3.2)) find

$$f_R = \frac{1}{\pi G} \left[ \mathbf{B} \times \text{curl} \mathbf{B} - \frac{1}{c} \mathbf{B} \times \frac{\partial \mathbf{F}}{\partial t} \right]. \quad (7.9)$$

We then write

$$-\frac{1}{c} \mathbf{B} \times \dot{\mathbf{F}} = \frac{1}{c} \dot{\mathbf{B}} \times \mathbf{F} - \frac{1}{c} \frac{\partial}{\partial t} (\mathbf{B} \times \mathbf{F}), \quad (7.10)$$

where the time average of the last term on the r.h.s. vanishes. For the first term on the r.h.s. we have from (7.8)

$$\frac{1}{c} \dot{\mathbf{B}} \times \mathbf{F} = \mathbf{F} \times \text{curl} \mathbf{F}, \quad (7.11)$$

therefore

$$f_R = \frac{1}{\pi G} [\mathbf{B} \times \text{curl} \mathbf{B} + \mathbf{F} \times \text{curl} \mathbf{F}]. \quad (7.12)$$

The gravitational radiation pressure therefore is four times larger than the corresponding one in electrodynamics.

Finally, we treat the interior solution of an incompressible fluid sphere. In Einstein's theory, this is known as Schwarzschild's interior solution. Our solution is somewhat different, because the pressure does not contribute to the gravitational field. A simple calculation then gives for the radial substratum velocity the following expression:

$$\left. \begin{aligned} v_r^2 &= \frac{2GM}{r}, \quad r > R, \\ v_r^2 &= \frac{GM}{r} \left[ 3 - \left( \frac{r}{R} \right)^2 \right], \quad r < R, \end{aligned} \right\} \quad (7.13)$$

where  $R$  is the radius of the sphere of mass  $M$ . If we introduce the Schwarzschild radius  $r_0 = 2GM/c^2$ , we obtain the interior line element:

$$ds^2 = \frac{dr^2}{1 - \frac{r_0}{2R} \left[ 3 - \left( \frac{r}{R} \right)^2 \right]}$$



$$+ r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) - c^2 \left[ 1 - \frac{r_0}{2R} \left[ 3 - \left( \frac{r}{R} \right)^2 \right] \right] dt^2. \quad (7.14)$$

## 8. Avoidance of the Singularities and the Causality-Violating Solutions

The singularity problem and its avoidance in the presented theory, as opposed to Einstein's theory, can most easily be studied for the spherically symmetric field. We first take the exterior solution of Einstein's theory, which is the same as in our theory, and is given by Schwarzschild's line element (6.2). According to our theory, the square of the ratio of the substratum velocity to the velocity of light is

$$\frac{v^2}{c^2} = \frac{2GM}{c^2 r} = \frac{r_0}{r}. \quad (8.1)$$

Therefore, the substratum velocity becomes superluminal for  $r < r_0$ . The possibility of a superluminal velocity is, of course, a consequence of the basic assumption that the substratum obeys a non-relativistic law of motion.

In the Lorentz-Poincaré ether interpretation of special relativity, a body moving with an absolute velocity  $v$  against the substratum suffers a true physical deformation. This deformation follows from the form of the electric scalar and vector potentials,  $\varphi$  and  $\mathbf{a}$ , of the electrodynamic forces holding the body together. If in a state of static equilibrium, and in case the velocity is along the  $x$ -axis, the equations for  $\varphi$  and  $\mathbf{a}$  in a system at rest with the ether are [4]

$$\left. \begin{aligned} \left( 1 - \frac{v^2}{c^2} \right) \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} &= -4\pi \varrho, \\ \left( 1 - \frac{v^2}{c^2} \right) \frac{\partial^2 \mathbf{a}}{\partial x^2} + \frac{\partial^2 \mathbf{a}}{\partial y^2} + \frac{\partial^2 \mathbf{a}}{\partial z^2} &= -(4\pi/c) \mathbf{j}, \end{aligned} \right\} \quad (8.2)$$

where  $\varrho$  and  $\mathbf{j}$  are the electric charge and current density within the body. It follows that by making everywhere, including in the sources, the replacement

$$x \rightarrow x \sqrt{1 - v^2/c^2}, \quad (8.3)$$

eqs. (8.2) go over into

$$\begin{aligned} \nabla^2 \varphi &= -4\pi \varrho, \\ \nabla^2 \mathbf{a} &= -(4\pi/c) \mathbf{j}, \end{aligned} \quad (8.4)$$

and which are the same as the equations if the body would be at rest in the substratum. The replacement (8.3) expressing the familiar Lorentz contraction, must also be made for a measuring apparatus moving along the body with the same velocity, and which thereby makes the contraction unobservable. As a consequence of the contraction, all clocks moving with the same absolute velocity go slower by the same factor  $\sqrt{1 - v^2/c^2}$ , and the velocity of light would appear isotropic and equal to  $c$ , with the anisotropy being cancelled out, due to the fact that in reality one can only measure the to and fro velocity. The Eqs. (8.2) can therefore be seen as the dynamic substratum interpretation of special relativity [4], and they tell us that a body with a velocity approaching the velocity of light would become unstable in reaching this limit, because no static equilibrium is possible for  $v > c$ . In the Lorentz-Poincaré interpretation of special relativity, this would mean that in the vicinity of  $v = c$ , special relativity would break down, and a body accelerated to  $v = c$  would in reaching this limit assume a very large mass, but not an infinite mass.

In a stationary gravitational field  $v > c$  always if

$$-2\Phi/c^2 > 1. \quad (8.5)$$

Returning to (8.1), this means that since for  $r < r_0$ ,  $v > c$ , matter placed within the Schwarzschild radius would become unstable. To understand how this is going to happen in greater detail, we turn to the interior solution, with the substratum velocity given by (7.13). There  $v > c$  happens if

$$v^2/c^2 = (r_0/2R) [3 - (r/R)^2] \geq 1. \quad (8.6)$$

Putting  $z = v^2/c^2$ ,  $r/R = y$ ,  $R/r_0 = x$ , we can write (8.6) as follows:

$$2xz = 3 - y^2. \quad (8.7)$$

The cut  $z = 1$  through this surface is the curve which determines the location of  $v = c$ :

$$y = \sqrt{3 - 2x}. \quad (8.8)$$

In particular  $y = 0$  for  $x = 3/2$ , which means that if

during a gravitational collapse when  $R = (3/2)r_0$ , the substratum velocity reaches  $v = c$  first in the center of the collapsing body and where matter begins to become unstable. If  $y = 1$  then  $x = 1$ , and  $v = c$  is reached at the surface of the collapsing body with  $v > c$  inside the entire body. In this situation, the radius of the body is equal to the Schwarzschild radius. Unlike in Einstein's theory, there is no singularity and there will be no black holes.

The disintegration of matter predicted in regions where the substratum velocity exceeds the velocity of light may perhaps explain the large amount of energy released by quasars, a phenomenon difficult to reconcile with the known laws of physics.

The paradox of travel back in time by passing through a rotating black hole, the other oddity of Einstein's theory, may have the following resolution: In the Lorentz-Poincaré interpretation, time never reverses its direction but objects suffer deformations. A clock representing a simple mechanism is a spinning top. In approaching a rotating black hole, a torque is exerted on the top by the Magnus-type force, slowing it down and eventually changing its direction of rotation. In leaving the gravitational field of the rotating black hole, the process would be reversed, and thereafter the top would be spinning in its original sense. The running in the opposite direction, while in the gravitational field of the rotating black hole, results in a negative phase shift, which would lead to a reading on a dial attached to the clock, giving the appearance that the clock left the field of the rotating black hole at an earlier time than when it entered this field\*.

It is easy to show what the change in the rate of small clocks is:

The intrinsic equation of motion for a spinning top is

$$\theta \frac{d\omega_c}{dt} = \tau, \quad (8.9)$$

where  $\theta$  is the moment of inertia of the top,  $\omega_c$  its angular velocity vector, and  $\tau$  the torque exerted

on it by the Magnus force. If  $m$  is the mass and  $r$  is the radius of the top, one has by order of magnitude

$$\begin{aligned} \theta &\approx m r^2, \\ \tau &\approx \left( \frac{4m}{c} \right) \left( \frac{\partial V}{\partial \mathbf{r}} \mathbf{r} \times \text{curl} \mathbf{A} \right) \times \mathbf{r}. \end{aligned} \quad (8.10)$$

With  $\partial V / \partial \mathbf{r} \approx \omega_c$  we, therefore, have

$$\left| \frac{1}{\omega_c} \frac{d\omega_c}{dt} \right| \approx \frac{4}{c} \left| \text{curl} \mathbf{A} \right|. \quad (8.11)$$

The change in the rate of the clock is, therefore, independent of its size and speed.

We may apply this result for a top approaching the surface of a spinning spherical mass of radius  $r_0 \sim GM/c^2$  and for which  $J \sim M r_0 c$ . This is qualitatively the situation resembling a clock approaching a Kerr black hole in general relativity. In the equatorial plane for  $r \sim r_0$  we have according to (7.2)  $|\text{curl} \mathbf{A}| \sim c^2/r_0$ , hence

$$\left| \frac{1}{\omega_c} \frac{d\omega_c}{dt} \right| \sim \frac{c}{r_0}. \quad (8.12)$$

The characteristic time  $t_0$  to change the rotational speed of the top, therefore, is

$$t_0 \sim r_0 / c. \quad (8.13)$$

In the equation of motion for the substratum (or ether), we had always omitted a pressure term. In making a simplified model of the ether, we may add a scalar pressure. The equation of motion for the substratum would then be (for  $\mathbf{A} = 0$ )

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = - \frac{1}{\varrho_e} \nabla p_e - \nabla \Phi \quad (8.14)$$

( $\varrho_e$  and  $p_e$  would be the density and pressure of the ether). This equation would have to be supplemented by the equation of continuity and an equation of state:

$$\begin{aligned} \frac{\partial \varrho_e}{\partial t} + \text{div}(\varrho_e \mathbf{v}) &= 0, \\ p_e &= \text{const } \varrho_e^\gamma, \end{aligned} \quad (8.15)$$

where  $\gamma = c_p / c_v$  would be the specific heat ratio of the ether. From the equation of state would follow

\* It is important to note that our conclusion only applies to a clock which (like a top) is a reversible mechanism. That the travel back in time paradox of general relativity may have its resolution in considering irreversible processes has been conjectured by von Weizsäcker [15], but it is difficult to see how this can be done in Einstein's theory.

a velocity for longitudinal compressional ether waves:

$$c^2 = \frac{dp_e}{d\rho_e} = \text{const } \gamma \rho_e^{\gamma-1} = \gamma \frac{p_e}{\rho_e}. \quad (8.16)$$

We must keep in mind that this can be only a very crude model since the ether waves as we know from electromagnetism, are transverse. In spite of this deficiency, we will nevertheless use this model to make some interesting speculations. If the ether behaves like a gas, then in accordance with (8.16) the velocity of light would there take the velocity of sound.

If this oversimplified model is capable of simulating physical reality, the ether flow, after having reached superluminal velocity  $v > c$ , can, in passing through a shock front, become discontinuously subluminal, accompanied by a rise in its entropy. In gasdynamics a shock discontinuity always occurs if the exact Riemann solution for finite amplitude waves becomes multivalued. In addition, a shock discontinuity can also connect regions of supersonic and subsonic flow. The same situation may arise whenever the ether velocity becomes superluminal and thereafter again becomes subluminal. In our theory, where a non-Euclidean metric is only a substitute description for real physical deformations of bodies in an otherwise Euclidean space, the occurrence of a shock discontinuity would manifest itself in a discontinuity of the substitute metric.

In gasdynamics, the occurrence of a shock discontinuity is accompanied by the emission of sound. In the flowing ether a shock discontinuity from superluminal to subluminal velocities would be accompanied by the emission of ether waves, that is light. Since a large astronomical body, having suffered a gravitational collapse inside the Schwarzschild radius, has a region where the ether velocity is superluminal, it could become subluminal by a shock transition.

The inflowing ether, attracted by a massive body and after having passed through a shock discontinuity, would thereafter leave the gravitational field of the body with a reduced outward velocity. At a distance larger than the one where the discontinuity occurs, the average square of the ether velocity,  $((v_{\text{in}}^2 + v_{\text{out}}^2)/2)$ , which otherwise determines the Schwarzschild metric, would appear to be reduced. This reduction would give an outside observer the impression that the body moves away at a velocity

which is equal to the velocity of the center of mass of the inflowing and outflowing ether. Therefore, if the discontinuity reduces the velocity of the inflowing ether by the amount  $\Delta v$ , resulting in a reduction of the outflowing ether by the same amount, a region at a distance larger than the discontinuity appears to move away with the velocity  $(1/2)\Delta v$ , resulting in a large red shift

$$\Delta v/v = -\Delta v/2c. \quad (8.17)$$

This could perhaps explain the observed anomalous red shifts of quasars, sometimes much larger than those of nearby galaxies. A discontinuity in the substratum velocity may also lead to the appearance of superluminal velocities, observed for quasar fragments and difficult to reconcile with the explanation that this phenomenon is an optical illusion.

From (8.14) and (8.15) follows the equation of motion for the compressible ether model in a spherical symmetric gravitational field:

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} + \frac{1}{\gamma-1} \frac{\partial c^2}{\partial r} = -\frac{\partial \Phi}{\partial r}. \quad (8.18)$$

The observed local Lorentz-invariance in a local inertial frame of reference would imply that

$$\frac{\partial c^2}{\partial r} = 0, \quad (8.19)$$

and which is possible only for  $\gamma = 1$ . If we consider the ether as a quasielastic fluid composed of discrete elements, the specific heat ratio is related to the number of degrees of freedom  $f$  of those elements, by

$$\gamma = \frac{2+f}{f}. \quad (8.20)$$

To obtain  $\gamma = 1$  requires  $f = \infty$ , and it therefore follows that the ether would behave like a continuum. For  $\gamma = 1$ , however, there would be no way for the ether to develop a shock discontinuity, and the "atoms" of the ether are therefore likely to have a very large, albeit finite number of degrees of freedom. A possible structure of these "ether atoms" might be quantized vortices in the superfluid substratum, and consisting of many Planckions. A medium possessing a microscopic structure made up of small vortices could also transmit transverse waves having the same properties as electromagnetic waves, as it was shown by Sir William Thomson about 100 years ago [16].

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